

Questions

22M.2.AHL.TZ1.10

Consider the function $fx = \sqrt{x^2 - 1}$, where $1 \leq x \leq 2$.

The curve $y = f(x)$ is rotated 2π about the y -axis to form a solid of revolution that is used to model a water container.

At $t = 0$, the container is empty. Water is then added to the container at a constant rate of $0.4 \text{ m}^3 \text{ s}^{-1}$.

a. Sketch the curve $y = fx$, clearly indicating the coordinates of the endpoints. [2]

b.i. Show that the inverse function of f is given by $f^{-1}x = \sqrt{x^2 + 1}$. [3]

b.ii. State the domain and range of f^{-1} . [2]

c.i.

Show that the volume, $V \text{ m}^3$, of water in the container when it is filled to a height of h metres is given by $V = \pi \frac{1}{3} h^3 + h$.

[3]

c.ii. Hence, determine the maximum volume of the container. [2]

d. Find the time it takes to fill the container to its maximum volume. [2]

e.

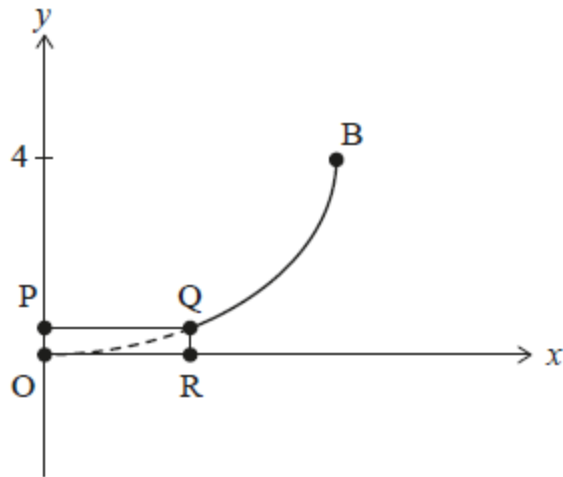
Find the rate of change of the height of the water when the container is filled to half its maximum volume.

[6]

22M.2.AHL.TZ2.6

The following diagram shows the curve $\frac{x^2}{36} + \frac{y - 4^2}{16} = 1$, where $h \leq y \leq 4$.

diagram not to scale



The curve from point Q to point B is rotated 360° about the y -axis to form the interior surface of a bowl. The rectangle OPQR, of height h cm, is rotated 360° about the y -axis to form a solid base.

The bowl is assumed to have negligible thickness.

Given that the interior volume of the bowl is to be 285 cm^3 , determine the height of the base.

17M.2.AHL.TZ2.H_2

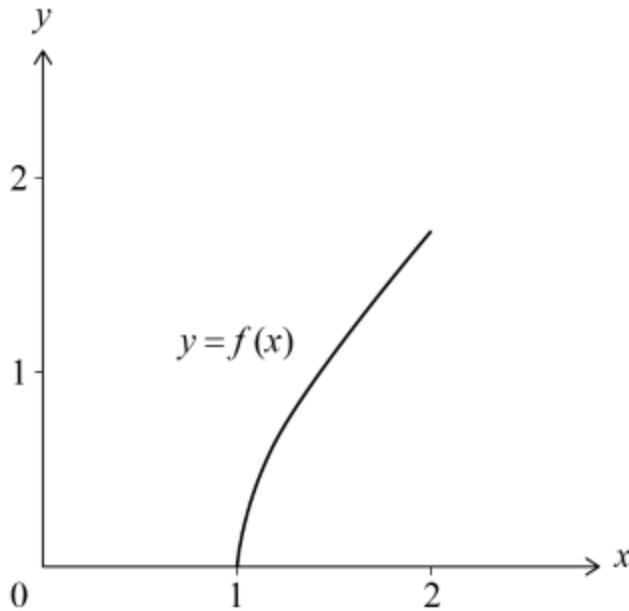
Consider the curve defined by the equation $4x^2 + y^2 = 7$.

Find the volume of the solid formed when the region bounded by the curve, the x -axis for $x \geq 0$ and the y -axis for $y \geq 0$ is rotated through 2π about the x -axis.

Markschemes

22M.2.AHL.TZ1.10

a.



correct shape (concave down) within the given domain $1 \leq x \leq 2$ **A1**

1, 0 and $2, \sqrt{3} = 2, 1.73$ **A1**

Note: The coordinates of endpoints may be seen on the graph or marked on the axes.

[2 marks]

b.i. interchanging x and y (seen anywhere) **M1** $x = \sqrt{y^2 - 1}$ $x^2 = y^2 - 1$ **A1**

$y = \sqrt{x^2 + 1}$ **A1** $f^{-1}x = \sqrt{x^2 + 1}$ **AG** **[3 marks]**

b.ii. $0 \leq x \leq \sqrt{3}$ OR domain $0, \sqrt{3} = 0, 1.73$ **A1**

$1 \leq y \leq 2$ OR $1 \leq f^{-1}x \leq 2$ OR range 1, 2 **A1** **[2 marks]**

c.i. attempt to substitute $x = \sqrt{y^2 + 1}$ into the correct volume formula **(M1)**

$$V = \pi \int_0^h \sqrt{y^2 + 1}^2 \, dy = \pi \int_0^h y^2 + 1 \, dy \quad \mathbf{A1} = \pi \frac{1}{3} y^3 + y \Big|_0^h \quad \mathbf{A1}$$

$$= \pi \frac{1}{3} h^3 + h \quad \mathbf{AG}$$

Award marks as appropriate for correct work using a different variable e.g.

$$\pi \int_0^h \sqrt{x^2 + 1}^2 dx$$

c.ii. attempt to substitute $h = \sqrt{3}$ = 1.732 ... into V $V = 10.8828 \dots$

$$V = 10.9 \text{ m}^3 = 2\sqrt{3}\pi \text{ m}^3$$

$$\text{d. time} = \frac{10.8828 \dots}{0.4} = \frac{2\sqrt{3}\pi}{0.4} = 27.207 \dots = 27.2 = 5\sqrt{3}\pi \text{ s}$$

e. attempt to find the height of the tank when $V = 5.4414 \dots = \sqrt{3}\pi$

$$\pi \frac{1}{3}h^3 + h = 5.4414 \dots = \sqrt{3}\pi \quad h = 1.1818 \dots$$

attempt to use the chain rule or differentiate $V = \pi \frac{1}{3}h^3 + h$ with respect to t

$$\frac{dh}{dt} = \frac{dV}{dV} \times \frac{dV}{dt} = \frac{1}{\pi h^2 + 1} \times \frac{dV}{dt} \quad \text{OR} \quad \frac{dV}{dt} = \pi h^2 + 1 \frac{dh}{dt}$$

attempt to substitute h and $\frac{dV}{dt} = 0.4$

$$\frac{dh}{dt} = \frac{0.4}{\pi (1.1818 \dots)^2 + 1} = 0.053124 \dots = 0.0531 \text{ ms}^{-1}$$

22M.2.AHL.TZ2.6

attempts to express x^2 in terms of y **(M1)**

$$V = \pi \int_h^4 361 - \frac{y-4^2}{16} dy \quad \mathbf{A1}$$

Note: Correct limits are required.

$$\text{Attempts to solve } \pi \int_h^4 361 - \frac{y-4^2}{16} dy = 285 \text{ for } h \quad \mathbf{(M1)}$$

Note: Award **M1** for attempting to solve $36\pi \frac{h^3}{48} - \frac{h^2}{4} + \frac{8}{3} = 285$ or equivalent for h .

$$h = 0.7926 \dots$$

$$h = 0.793 \text{ (cm)} \quad \mathbf{A2}$$

17M.2.AHL.TZ2.H_2

Use of $V = \pi \int_0^{\frac{\sqrt{7}}{2}} y^2 dx$

$$V = \pi \int_0^{\frac{\sqrt{7}}{2}} (7 - 4x^2) dx \quad \textbf{(M1)(A1)}$$

Note: Condone absence of limits or incorrect limits for **M** mark.

Do not condone absence of or multiples of π .

$$= 19.4 \left(= \frac{7\sqrt{7}\pi}{3} \right) \quad \textbf{A1}$$

[3 marks]